Oral Qualifying Exam Syllabus

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Major: Optimization theory

Convex Optimization

1. Unconstrained optimization [Ahm, §3]
    - Applications: least squares, detecting feasibility
    - Descent directions and first order optimality conditions
    - Second order optimality conditions
2. Convex sets (and functions) [Ahm, §4]
    - Definition and mid-point convexity and examples
    - Sublevel sets, quasiconvexity and epigraphs
    - Convex hull and Caratheodory's theorem
3. Separating hyperplane theorem, Farkas lemma and LP duality [Ahm, §5]
4. An application of LP strong duality to combinatorial optimization [Ahm, §6]
    - Bipartite matching and vertex covers
    - König's theorem
    - Totally unimodular matrices and integral polytopes
5. Convex functions [Ahm, §7]
    - Convex, concave, strictly convex, and strongly convex functions
    - First and second order characterizations of convex functions
    - Optimality conditions for convex problems
6. Convexity-preserving operations [§2 Par; Ahm, §8]
7. Convex envelopes, cardinality constrained optimization and LASSO [Ahm, §8]
8. LP, QP, QCQP, SOCP, SDP [Ahm, §9]
9. Applications of SDP in dynamical systems and combinatorics [Ahm, §10, 11]
10. Nonconvex quadratic optimization and its SDP relaxation, the S-Lemma. [Ahm, §12]
11. Computational complexity in numerical optimization. [Ahm, §13, 14]
12. Sum of squares programming and relaxations for polynomial optimization. [Ahm, §15]
13. Robust optimization. [Ahm, §16]
14. Convex relaxations for NP-hard problems, approximation algorithms. [Ahm, §17, 18]

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Algorithmic Algebraic Geometry

1. Solving polynomial equations
    - Solving Polynomial Systems by Elimination [CLO05, §2.1]
    - Solving Equations via Eigenvalues and Eigenvectors [CLO05, §2.5]
2. Resultants
    - The resultant of two polynomials and multipolynomial resultant [CLO05, §3.1, 3.2]
    - Resultants, discriminants, applications [Par, §6]
    - Properties of resultants and computing them [CLO05, §3.3, 3.4]
    - Solving Equations via Resultants [CLO05, §3.5]
3. Polytopes, Resultants, and Equations
    - Geometry of polytopes [CLO05, §7.1]
    - Minkowski Sums and Mixed Volumes [CLO05, §7.4]
    - Bernstein's Theorem [CLO05, §7.5]
    - Computing Resultants and Solving Equations [CLO05, §7.6]
4. Binomial Equations, Newton Polytopes, The Bézout and BKK Bounds, Application to Nash Equilibria [Par, §9]
5. Nonegativity, SOS, Semidefinite Programming [Par, §10]

Minor: Probability Theory

1. Measure theory
    - Probability Spaces [Dur19, §1.1]
    - Distributions [Dur19, §1.2]
    - Random Variables [Dur19, §1.3]
    - Integration [Dur19, §1.4]
    - Expected value [Dur19, §1.6]
2. Laws of Large Numbers
    - Independence [Dur19, §2.1]
    - Weak Laws of Large Numbers [Dur19, §2.2]
    - Borel-Cantelli Lemmas [Dur19, §2.3]
    - Strong Law of Large Numbers [Dur19, §2.4]
3. Central Limit Theorems
    - Weak Convergence [Dur19, §3.2]
    - Characteristic Functions [Dur19, §3.3]
    - Central Limit Theorems [Dur19, §3.4]
4. Concentration inequalities
    - Chernoff bound [Wai19, §2.1.1]
    - Sub-Gaussian variables and Hoeffding bounds [Wai19, §2.1.2]
    - Sub-exponential variables and Bernstein bounds [Wai19, §2.1.3]

References

[CLO05] D. A. Cox, J. B. Little, and D. O'Shea. *Using Algebraic Geometry*. Second. Vol. 185. Graduate Texts in Mathematics. Springer, 2005. URL: `https://doi.org/10.1007/b138611`.

[Dur19] R. Durrett. *Probability: theory and examples*. Vol. 49. Cambridge university press, 2019. URL: `https://services.math.duke.edu/~rtd/PTE/PTE5_011119.pdf`.

[Wai19] M. J. Wainwright. *High-Dimensional Statistics: A Non-Asymptotic Viewpoint*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 2019.

[Ahm] A. A. Ahmadi. *Lecture notes on Convex and Conic Optimization*. URL: `https://aaa.princeton.edu/orf523`.

[Par] P. Parrilo. *Lecture notes on Algebraic Techniques And Semidefinite Optimization*. URL: `https://ocw.mit.edu/courses/6-972-algebraic-techniques-and-semidefinite-optimization-spring-2006/`.